Stochastic and deterministic population processes: From branching to the transport equation.

Marek Kimmel, Rice University, Houston, TX

Warsaw 2009

There exist two major traditions of mathematical description of population dynamics, stochastic and deterministic. As it is known, stochastic description is needed in the case when the number of individuals in the population is small, whereas for large populations the law of large numbers makes expected values sufficiently good descriptors. Following this principle, and in agreement with intuition, modeling with a multitype Markov age-dependent branching processes (bp) can be replaced for large populations by modeling with a system of linear autonomous differential equations. However, the analogies are not always obvious. For example, in the case of populations with structure, the deterministic paradigm leads to the partial differential equations of transport type. The corresponding bp has to have a continuum of types and its mathematical description requires application of stochastic point processes and corresponding probability generating functionals. In the talk, following necessary preliminaries, we will consider a class of continuum-type bp, the expected values of which, under suitable regularity assumptions, satisfy transport-type equations. Moreover, arguably, the bp formalism enables to choose the right form of boundary conditions for the problems in which the type space has a mixed, discrete-continuous, character. Examples include the continuous-maturation model of blood cell production.